Session Types for the Concurrent Composition of Interactive Differential Privacy

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1. Differential Privacy

Motivation of Differential Privacy

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- ✗ How much does Jane Doe earn?

The answer to a privacy-preserving query does not depend on any individual in particular.

Formal Definition of Differentially Privacy

Definition (Dwork et al. 2006, Definition 1)

A probabilistic algorithm M is (ϵ, δ)-*differentially private* if, for any pair of adjacent databases D and D', the following condition holds:

$$M(D) \underset{(\epsilon,\delta)}{\approx} M(D')$$

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- ϵ: privacy loss (the smaller the better),
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Theorem

Differential privacy is compositional.

Differential Privacy and Noise Addition

Theorem (Laplace mechanism)

If $q : \text{Data} \to \mathbb{R}$ *is a query such that for some* k

$$\forall D, D' \,.\, D \sim D' \Rightarrow |q(D) - q(D')| \leq k,$$

then the query

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If you know how sensitive a request is, you know how to make it privacy-preserving.

The Fuzz Calculus

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Definition

A function f between two metric spaces (X, d_X) and (Y, d_Y) is s-sensitive if for all x and x' in X, we have $d_Y\big(f(x), f(x')\big) \leq s \cdot d_X(x, x')$.

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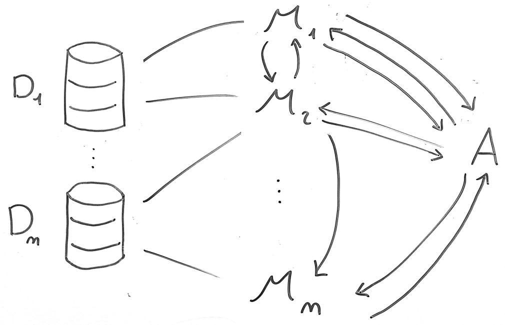
Theorem (Soundness)

If $[x : \sigma]_s \vdash f : \tau$*, then* $[\![f]\!]$ *is s-sensitive, and* $[\![f]\!] + \mathrm{noise}(s)$ *preserves DP.*

2. Interactive Differential Privacy

Interactive Mechanisms

What if we enable communication through multiple mechanisms, allowing, for example, the answer from one mechanism to be used to query another?



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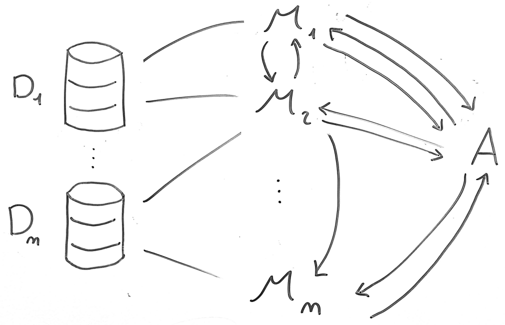

What would serve as the output of the mechanisms?

Interactive Differential Privacy

Definition (Vadhan and Wang 2021, Definition 1.6)

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- all the messages it receives during the interaction,
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Concurrent Composition of Interactive Differential Privacy

Theorem (Vadhan and Wang 2021, Theorem 1.8)

If interactive mechanisms $(M_1, \ldots, M_k)$ *are each* (ϵ, δ)*-differentially private, then their concurrent composition* $\mathrm{ConComp}(M_1, \ldots, M_k)$ *is* $\left(k\epsilon, \frac{e^{k\epsilon-1}}{e^{\epsilon-1}}\delta\right)$*-DP.*

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centralised DP : Fuzz :: interactive DP : ?

3. Process Calculi and Session Types

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Example

$$\big(k![1] . k?[x] . \dots\big) \parallel \big(k?[x] . k![x + x] . \dots\big)$$

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Theorem

- *Typing is preserved by reduction.*
- *A typable program never reduces into an error.*

4. Session Types for Interactive Differential Privacy

New Constructs for the π-calculus

We introduce two new constructs to the standard π-calculus:

- $\mathbf{Lap}_b(x)\,.\,P$ to sample a random number from the (discrete) Laplace distribution with parameter b and continue according to P,

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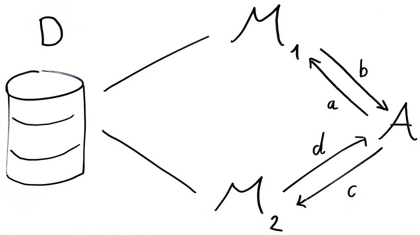
- $\mathbf{Lap}_b(x)\,.\,P$ to sample a random number from the (discrete) Laplace distribution with parameter b and continue according to P,
- $*_n P$ for the replication of the process P n times (this serves as a as a partial replacement for recursive processes).

Example of a Concurrent Composition



Let M_1 and M_2 be two differentially private mechanisms.

$$M_i = k_i?(f) \,.\, \mathbf{Lap}_{1/\epsilon}?(r) \,.\, k_i![f(D) + r] \,.\, 0$$

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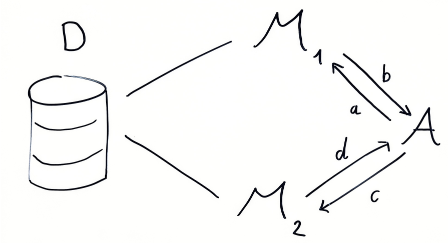

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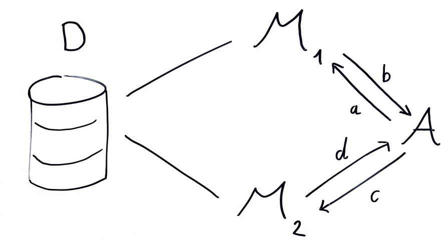

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$M_1 \parallel M_2$ is also differentially private, which means that it does not leak private information when interacting with *any* adversary.
For example, one possible adversary is

$$A = k_1![f] \,.\, k_1?(y_1) \,.\, k_2![g(y_1)] \,.\, k_2?(y_2) \,.\, \ldots \,.$$

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In practice, we use Fuzz as our expression language to benefit from its capability for sensitivity analysis in our typing rules.

Examples of Typing Rules

$$\frac{\Gamma \vdash e : \mathsf{Bool} \qquad \Gamma \vdash P \triangleright \Delta; (\epsilon_P, \delta_P) \qquad \Gamma \vdash Q \triangleright \Delta; (\epsilon_Q, \delta_Q)}{\Gamma \vdash \textbf{if } e \textbf{ then } P \textbf{ else } Q \triangleright \Delta; (0, 1)} \text{ [T-If]}$$

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Examples of Reduction Rules

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$$\frac{}{\mathbf{Lap}_b?(x) \,.\, P\left\{\xrightarrow[p_{n,b}]{\gamma_n} P[n/x]\right\}_{n\in\mathbf{Z}}} \text{[R-Lap]}$$

View of a process

The *trace* of the execution of P is the unique random variable $\mathrm{Trace}(P)$ such that if $P\left\{\xrightarrow[p_i]{t_i}{}^{*} P_i\right\}$, then $\Pr[\mathrm{Trace}(P) = t_i] = p_i$.

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Definition

The *view* of a process A interacting with a process M, is the following random variable: $\mathrm{View}(A \parallel M) = \mathrm{Left}\big(\mathrm{Trace}(A \parallel M)\big)$.

Differential Privacy as Approximate Trace Equivalence

We can use the same definition of interactive differential privacy as Vadhan, where View is formally defined as above.

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Lemma

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Hence, as all other rules are sound, our main theorem follows.

Theorem (Soundess)

If $\Gamma \vdash M \triangleright \Delta; (\epsilon, \delta)$*, then M is an* (ϵ, δ)*-differentially private process.*

5. Conclusion

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- reformulated interactive differential privacy in a formal operational way,
- defined typing rules for tracking interactive differential privacy, and
- provided examples, notably from Lyu (2022), demonstrating how privacy-preserving programs can be implemented in our calculus.

Future Work

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- define interactive differential privacy in terms of approximate bisimulation rather than approximate trace equivalence.

References I

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More Details on the Fuzz Language

Definition

A function f between two metric spaces (X, d_X) and (Y, d_Y) is s-sensitive if for all x and x' in X, we have $d_Y\big(f(x), f(x')\big) \leq s \cdot d_X(x, x')$.

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Types are interpreted as metric spaces:

- $[\![A \otimes B]\!] = [\![A]\!] \times [\![B]\!]$, and $[\![A \,\&\, B]\!] = [\![A]\!] \sqcup [\![B]\!]$,
- $[\![!_s A]\!] = \big(\pi_1([\![A]\!]), s \cdot \pi_2([\![A]\!])\big)$,
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Typing judgements have the form $[x_1 : A_1]_{s_1}, \ldots, [x_n : A_n]_{s_n} \vdash b : B$ and mean that $(x_1, \ldots, x_n) \mapsto [\![b]\!](x_1, \ldots, x_n)$ is a 1-sensitive function from $!_{s_1}[\![A_1]\!] \otimes \cdots \otimes !_{s_n}[\![A_n]\!]$ to $[\![B]\!]$.

Finite Replication and Recursive Processes

We permit finite process replication instead of recursive processes or arbitrary replication. This way, a process will never generate an infinite number of random numbers during its execution.

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Indeed, we aim to develop a formal framework for interactive differential privacy, rather than extending the existing notion.

- Vadhan and Wang (2021) generate binary strings before the interaction.
- Lyu (2022) explicitly bounds the number of interaction rounds.

$\rho \leftarrow \mathcal{L}(1/\epsilon)$;
for $i = 1, 2, \ldots, T$ **do**
  Receive the next query (f_i, τ_i);
  $\gamma_i \leftarrow \mathcal{L}(c/\epsilon)$;
  if $|f_i(X) - \tau_i| + \gamma_i \geq E + \rho$ **then**
    $v_i \leftarrow f(X) + \mathcal{L}(c/\epsilon)$;
    report (wrong, v_i);
    $t \leftarrow t + 1$;
    if $t = c$ **then halt**;
  else
    report pass
  end
end

Algorithm 1: Private Guess-and-Check Lyu 2022, Algorithm 1

```
SVT(c, E, N, D, k, a) =
  Lap(1/epsilon)?(rho);
  a.write 1;
  repeat N times
    k?(f, v);
    let t = a.read () in
    if t >= c then k![0] else
      Lap(c/epsilon)?(gamma);
      k![abs (f(D) - v) + gamma < E +
    end
    a.write (t + 1);
  end
```

Guess-and-Check (II)

Theorem

Given the environment $\Gamma = \{c : \mathrm{Nat}, E : \mathrm{Int}, N : \mathrm{Nat}, D : \mathrm{Data}, a : \mathrm{Cell}(\mathrm{Nat})\}$ *and the typing* $\Delta = \{k : *_N?\big((\mathrm{Data} \multimap \mathrm{Int}) \otimes \mathrm{Int}\big) . !\mathrm{Int} . \mathrm{end}\}$, *the following typing rule is sound:*

$$\frac{}{\Gamma \vdash \mathrm{SVT}(c, E, N, D, k, a) \triangleright \Delta; (3\epsilon, 0)} \text{ [T-SVT]} \tag{1}$$